A half de Sitter Holography

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01. Introduction- de Sitter Space

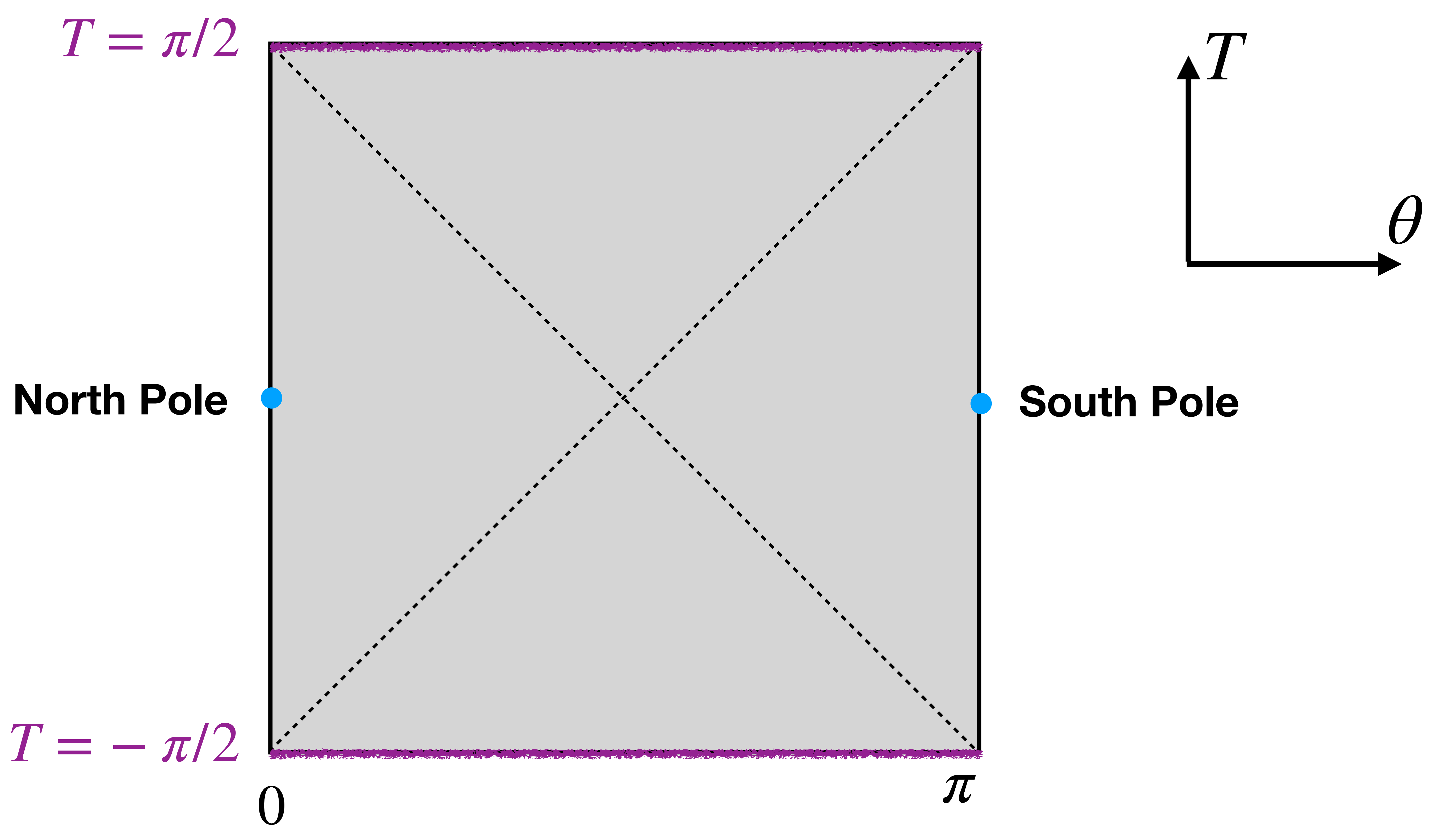
dS/CFT Correspondence

[Strominger 2001]

$$ds^2 = L^2\left(-dt^2 + \cosh^2 t\,(d\theta^2 + \sin^2\theta d\Omega_{d-1}^2)\right)$$

$$= \frac{L^2}{\cos^2 T}\left(-dT^2 + d\theta^2 + \sin^2\theta d\Omega_{d-1}^2\right)$$

$$S_{\rm dS} = \frac{\rm Area}{4\,G_{\rm N}} = \frac{L^{d-1}\,\Omega_{d-1}}{4\,G_{\rm N}} \equiv N$$

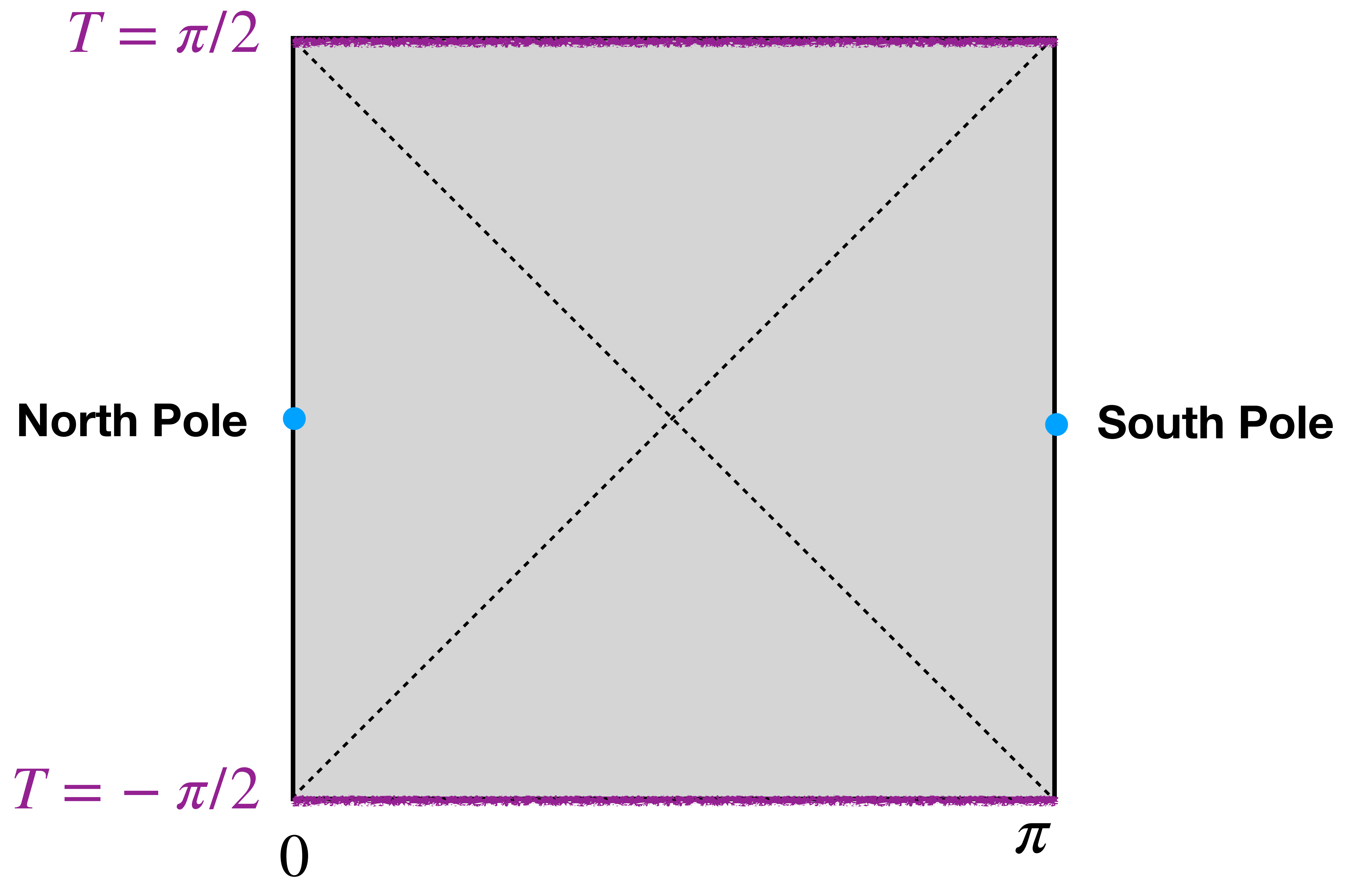


“Analytical continuation” of AdS/CFT correspondence

dS/CFT correspondence: gravity in asymptotically dS_{d+1} spacetime
a d-dimensional CFT living on the spacelike boundary at (timelike) future infinity of dS.

dS/CFT Correspondence

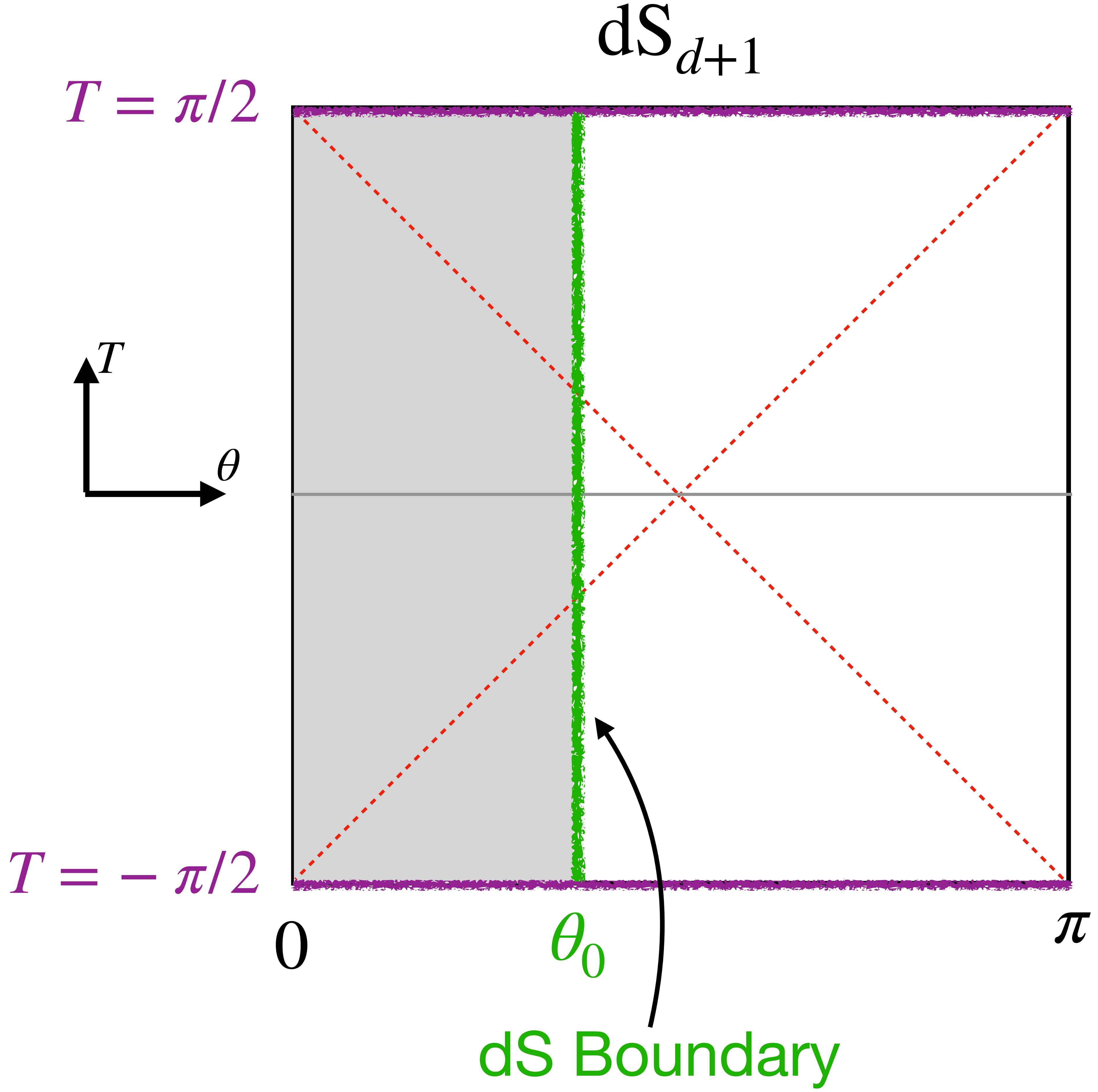
[Strominger 2001]

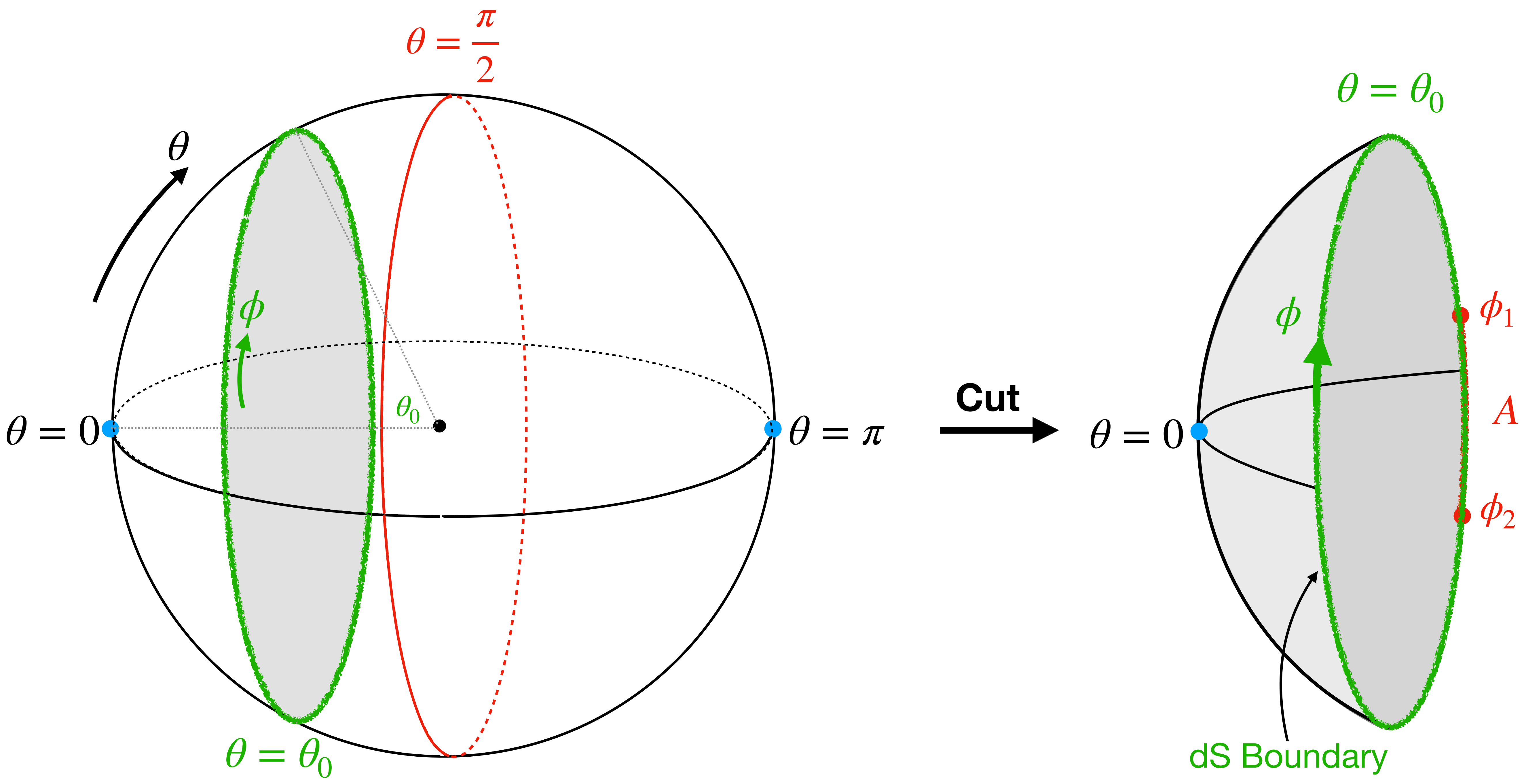


dS_4 [Anninos-Hartman-Strominger 2011]
dS_3 [Hikida,Nishioka,Takayanagi,Taki 2021]

- de Sitter is forbidden as supersymmetry solution to supergravity
- Dual CFT in dS/CFT is exotic, e.g., dS_3/CFT_2, pure imaginary central charge $c = i\frac{3L_{dS}}{2G_N}$

Topic Today: A half de Sitter space





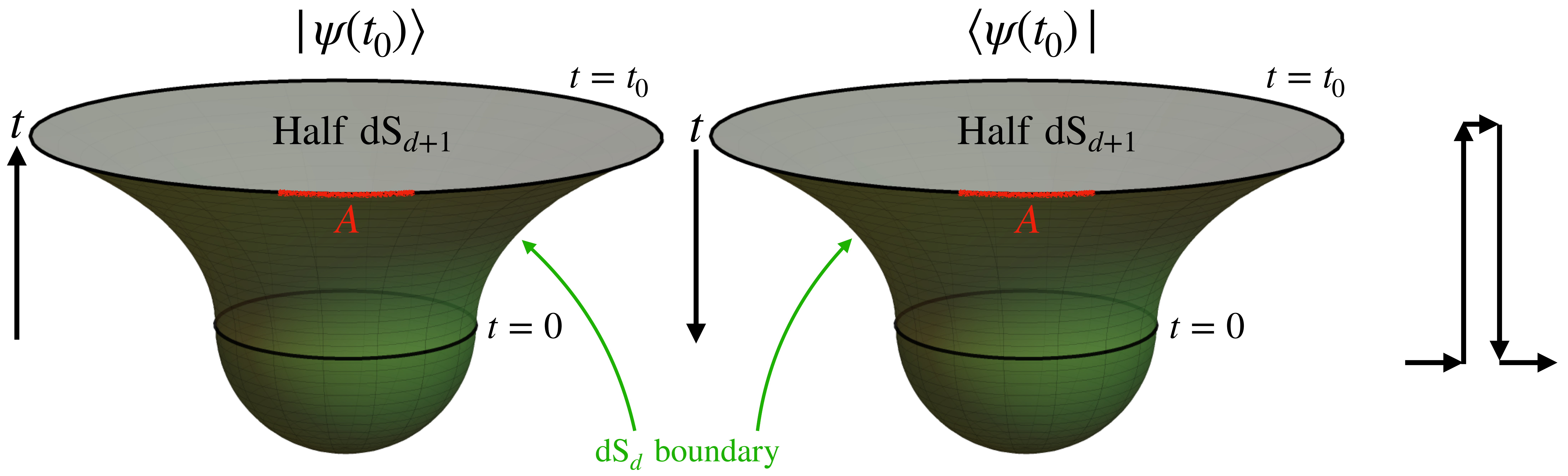
$$ds^2 = -dt^2 + \cosh^2 t\,(d\theta^2 + \sin^2\theta d\Omega_{d-1}^2)$$

d+1-dim de Sitter Bulk

$$ds^2 = -dt^2 + \cosh^2 t \sin^2\theta_0 d\Omega_{d-1}^2$$

d-dim de Sitter Boundary

Perspective 01: Schwinger-Keldysh Prescription



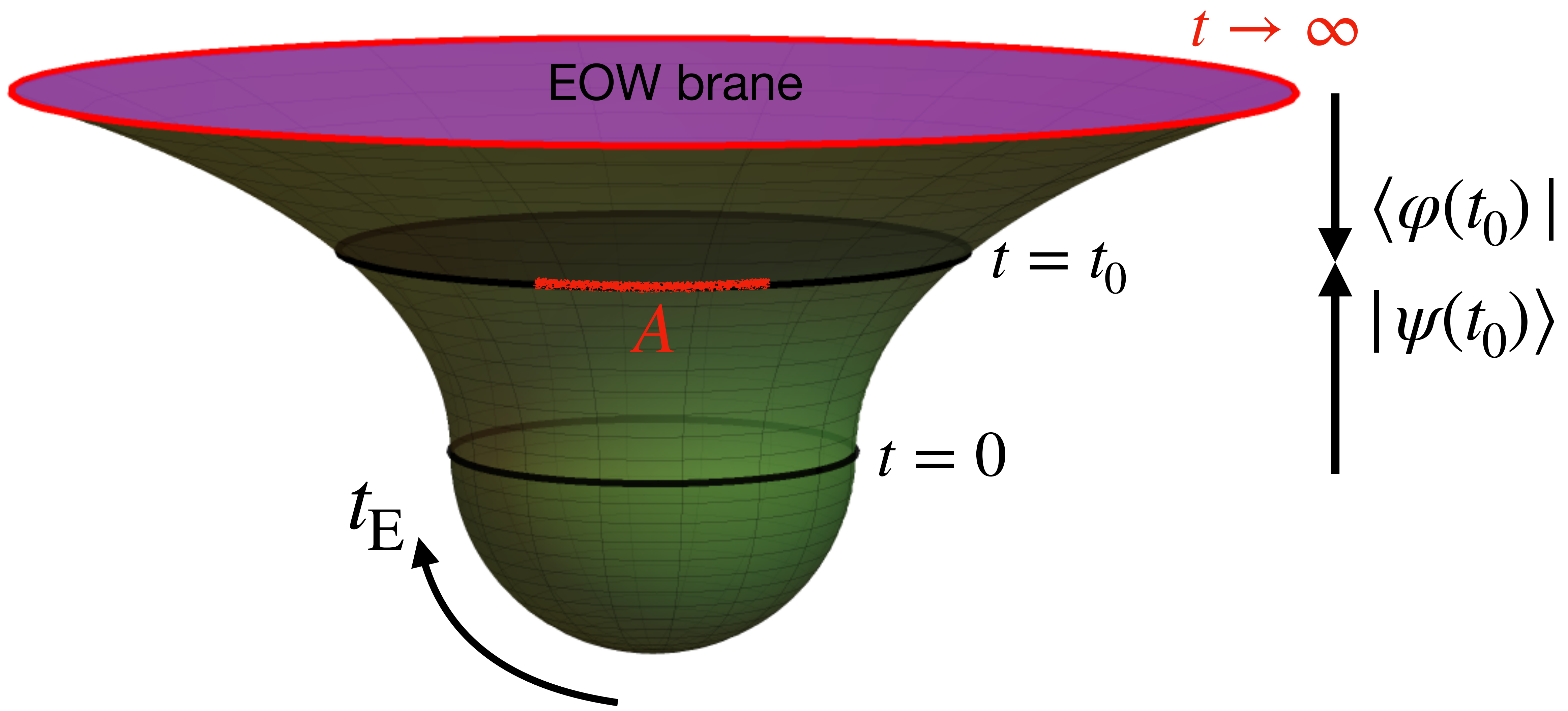
Holographic Entanglement Entropy

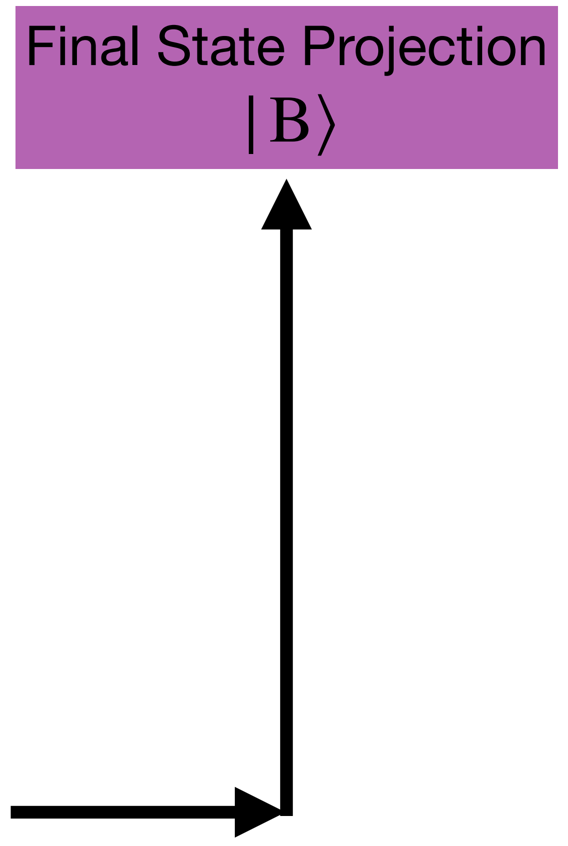
$$\rho_A = \mathrm{Tr}_B \left[\frac{|\psi\rangle\langle\psi|}{\langle\psi\,|\,\psi\rangle} \right]$$

$$S_A = \mathrm{Tr}\left[-\rho_A \log \rho_A\right]$$

Perspective 02: Final State Projection

Final state projection at future infinity





Reduced transition matrix

$$\tau_A = \mathrm{Tr}_B \left[\frac{|\psi\rangle \langle \varphi|}{\langle \psi | \varphi \rangle} \right]$$

It is not always Hermitian!

A generalization of entanglement entropy

Pseudo entropy $\quad S_A = \mathrm{Tr}\left[-\tau_A \log \tau_A\right]$

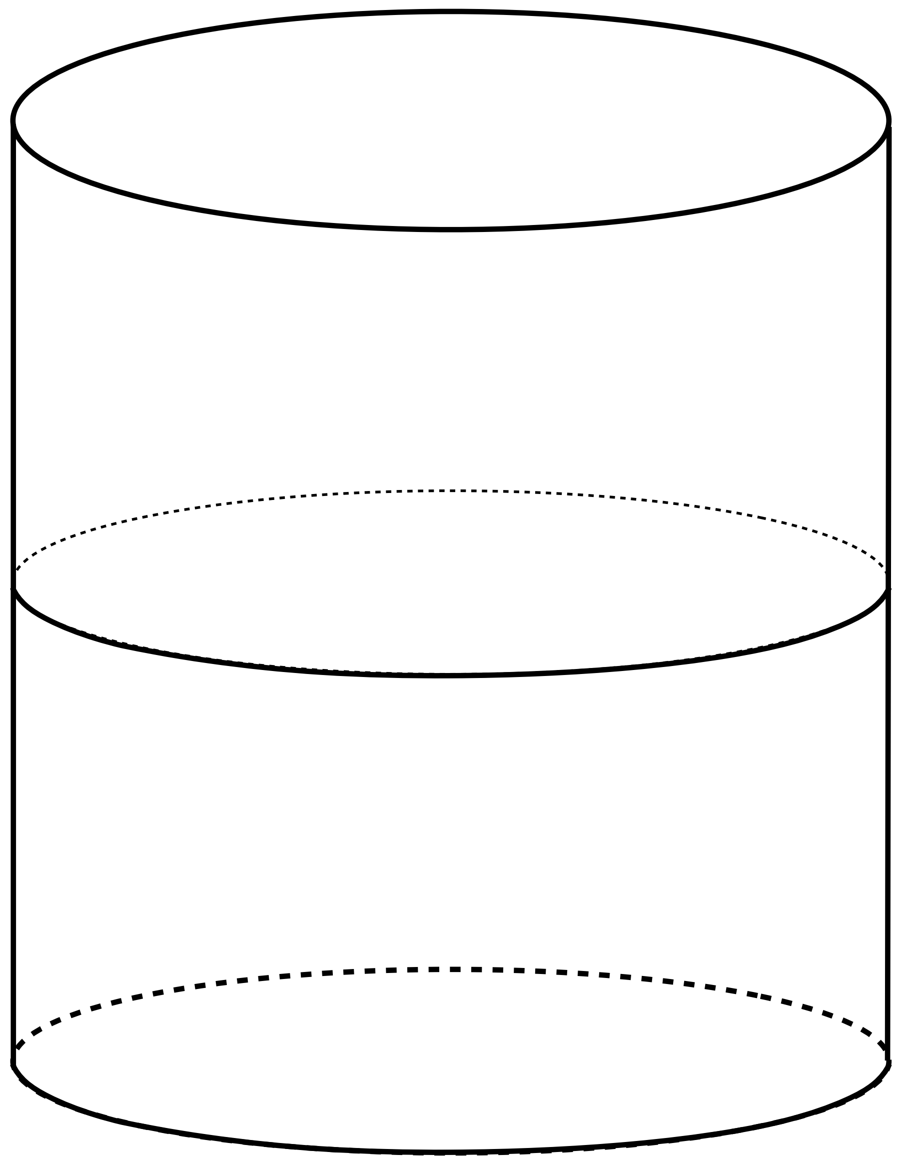
Complex Values!

Holographic Pseudo Entropy

[Nakata, Takayanagi, Taki, Tamaoka, Wei 2020]

Note: Its holographic dual is also given by the area of the minimal surface

02. CFT on dS space via AdS/CFT

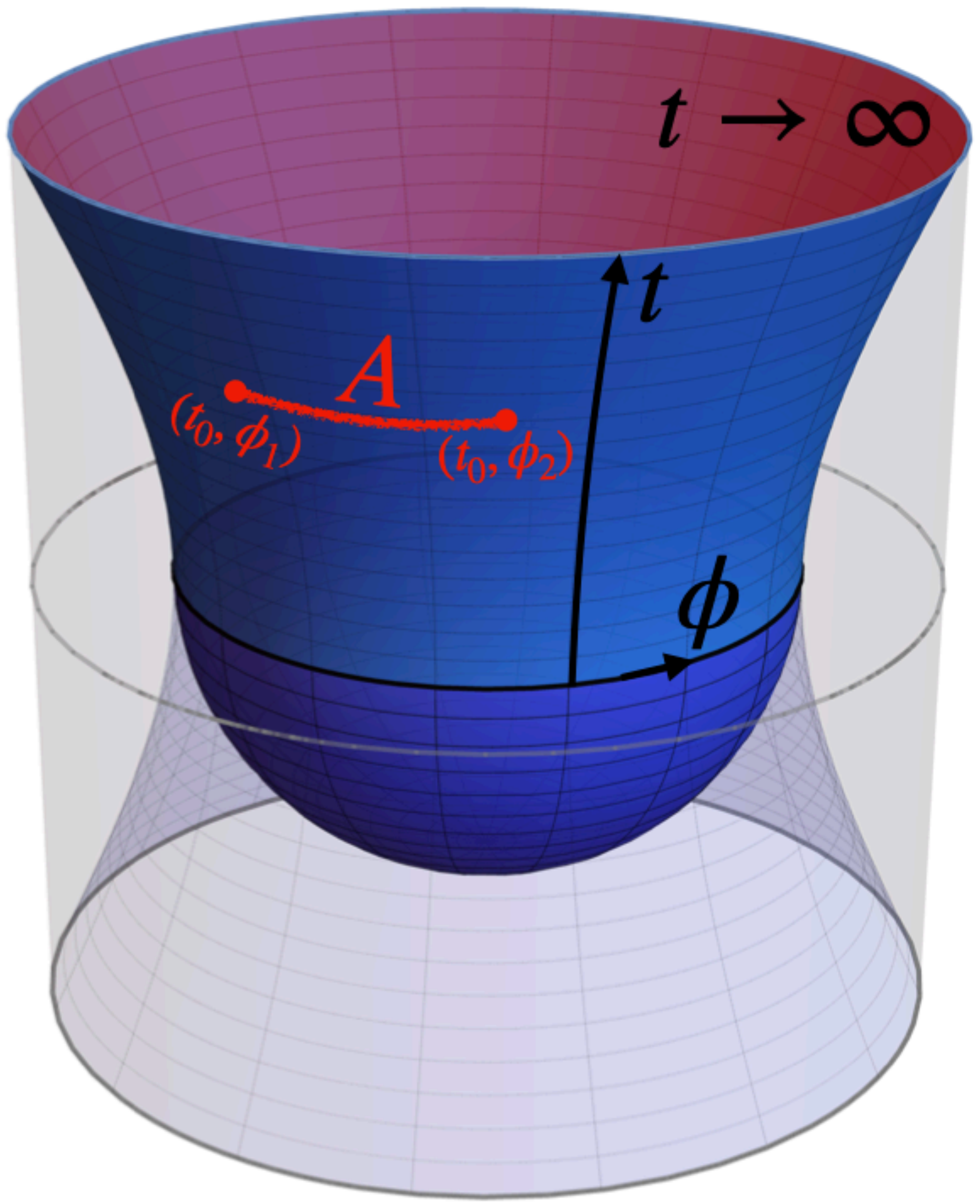


Perspective 01:Holographic Entanglement Entropy

Hartle-Hawking State

"No boundary proposal"

[“Wave Function of the Universe” 1983]



Lorentzian Evolution

Euclidean Evolution

AdS$_3$/CFT$_2$

$$S_A^{\text{con}} = \frac{c}{3} \log \left[\frac{2 \cosh t_0 \sin \frac{|\phi_1 - \phi_2|}{2}}{\epsilon} \right],$$

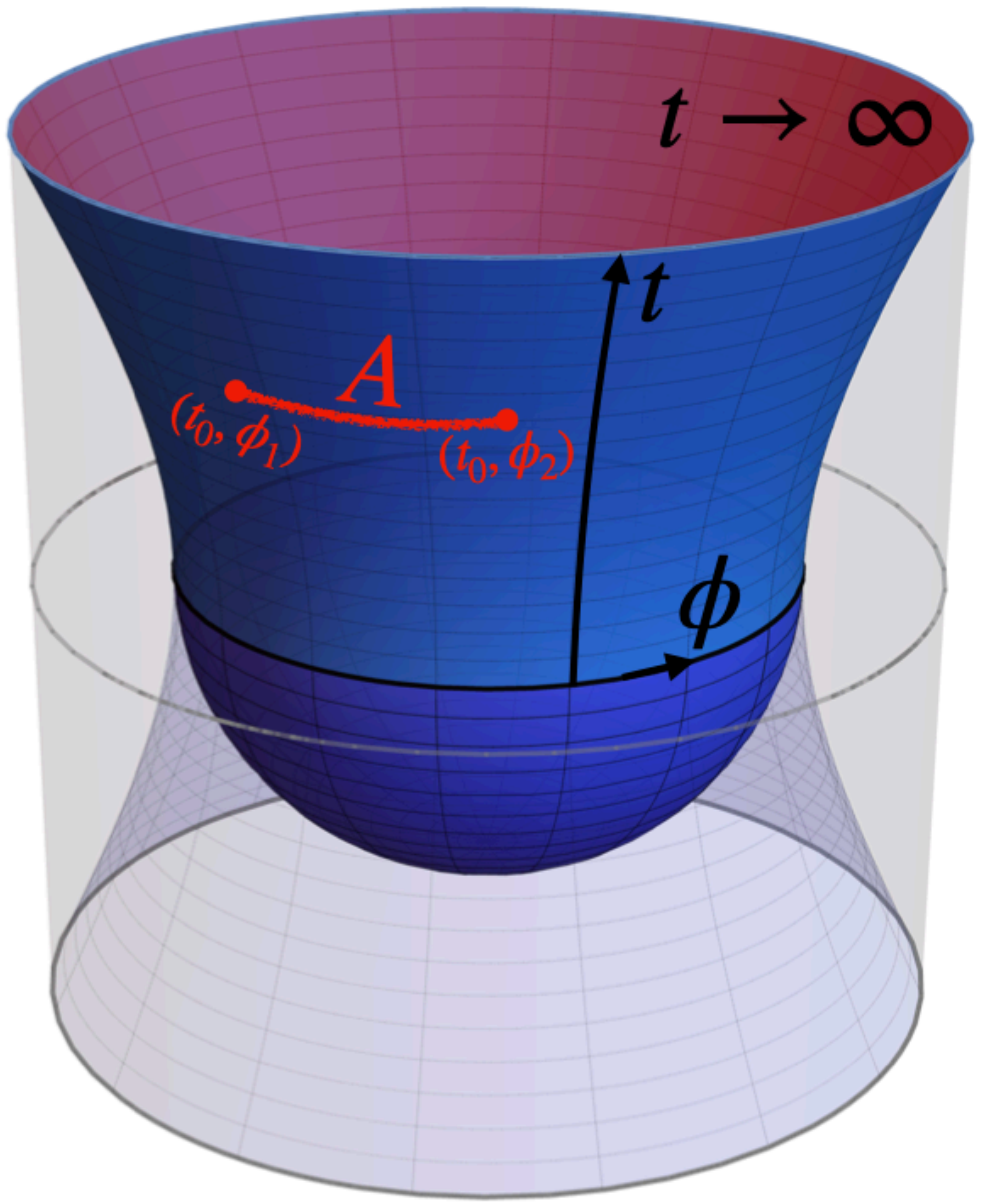
Linear growth at late times

Schwinger-Keldysh prescription

Future infinity plays no role!

Perspective 02: Holographic Pseudo Entropy

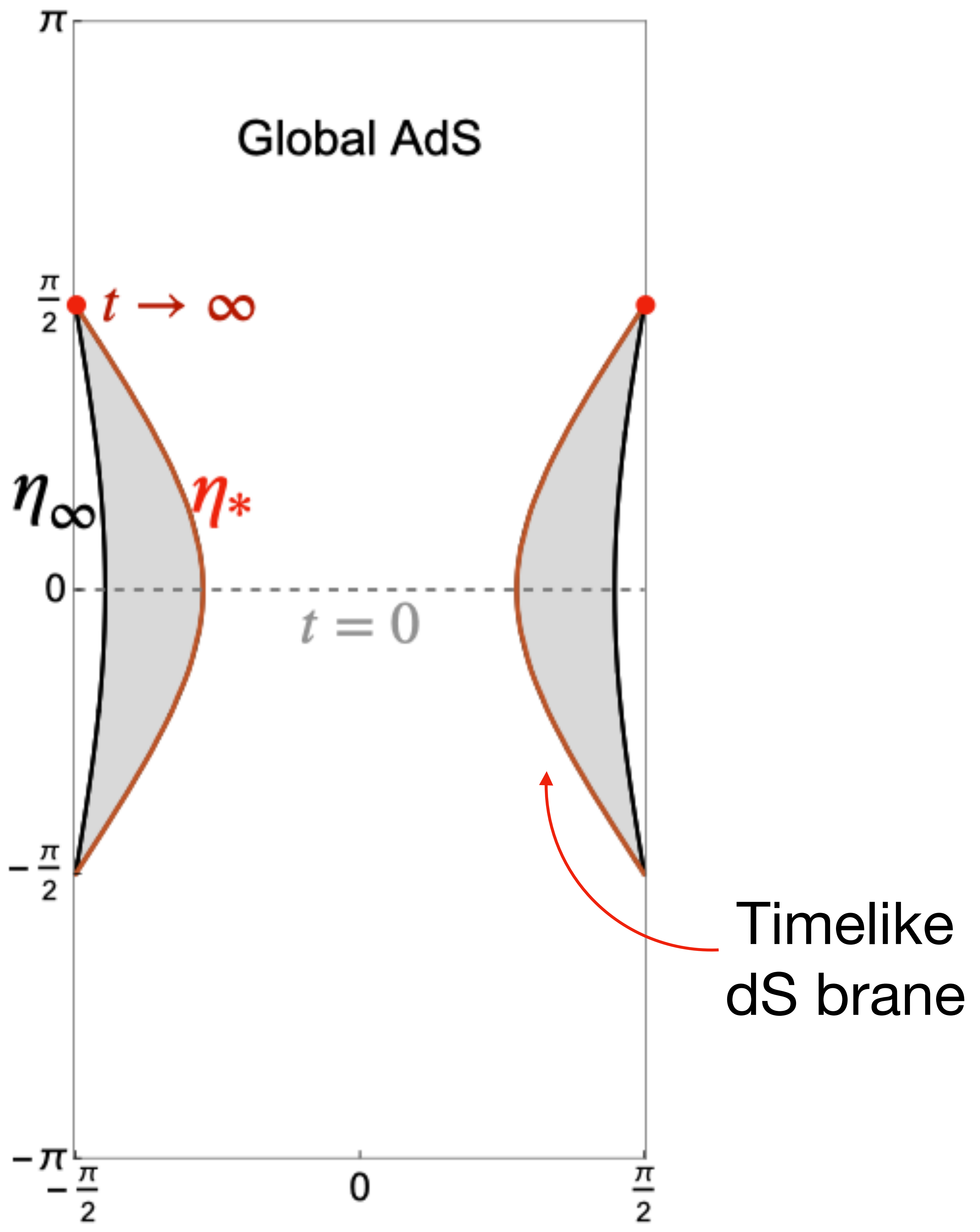
Boundary state



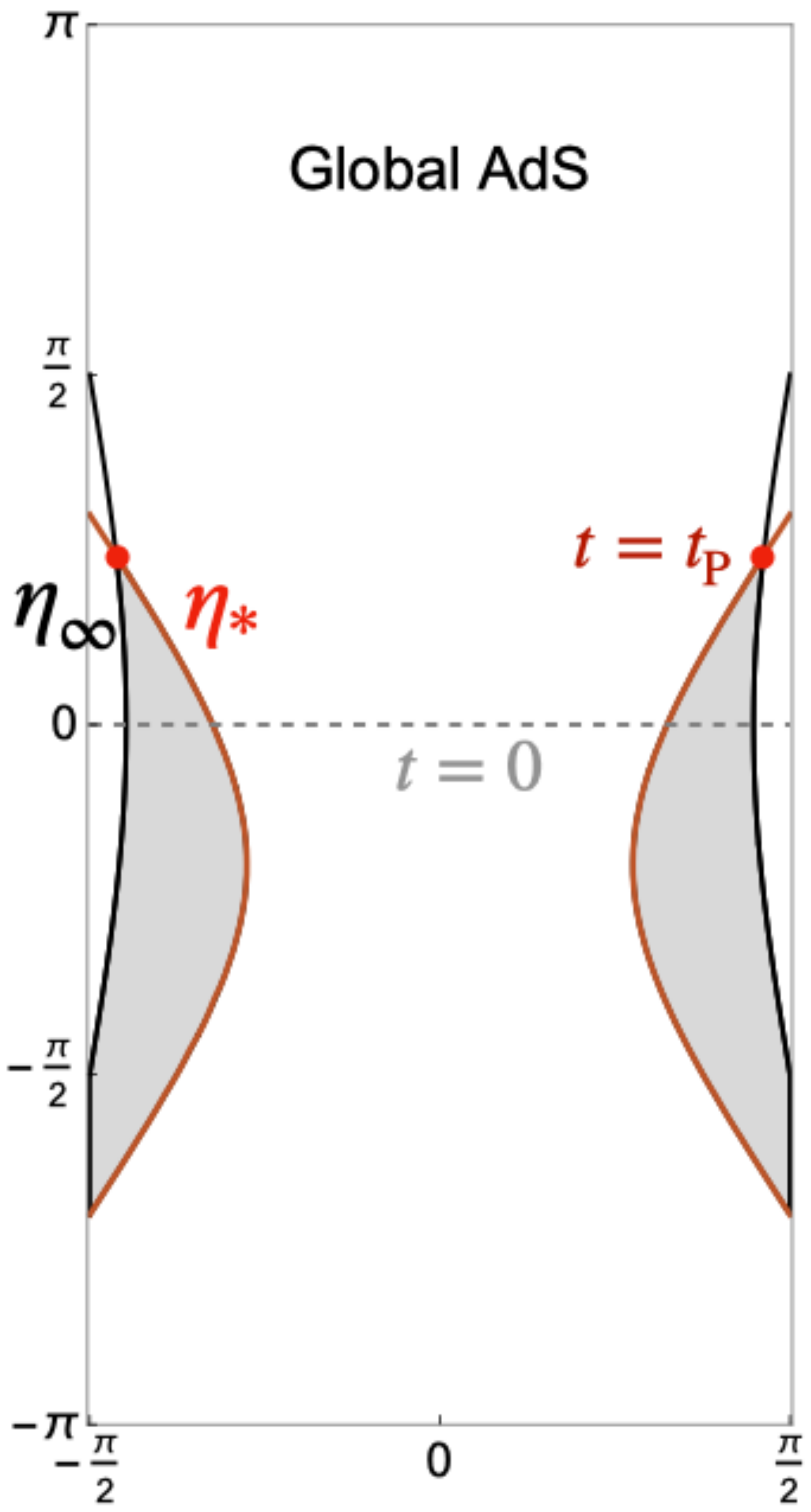
Final state projection
on the boundary

↓

End-of-the-World brane
in the bulk



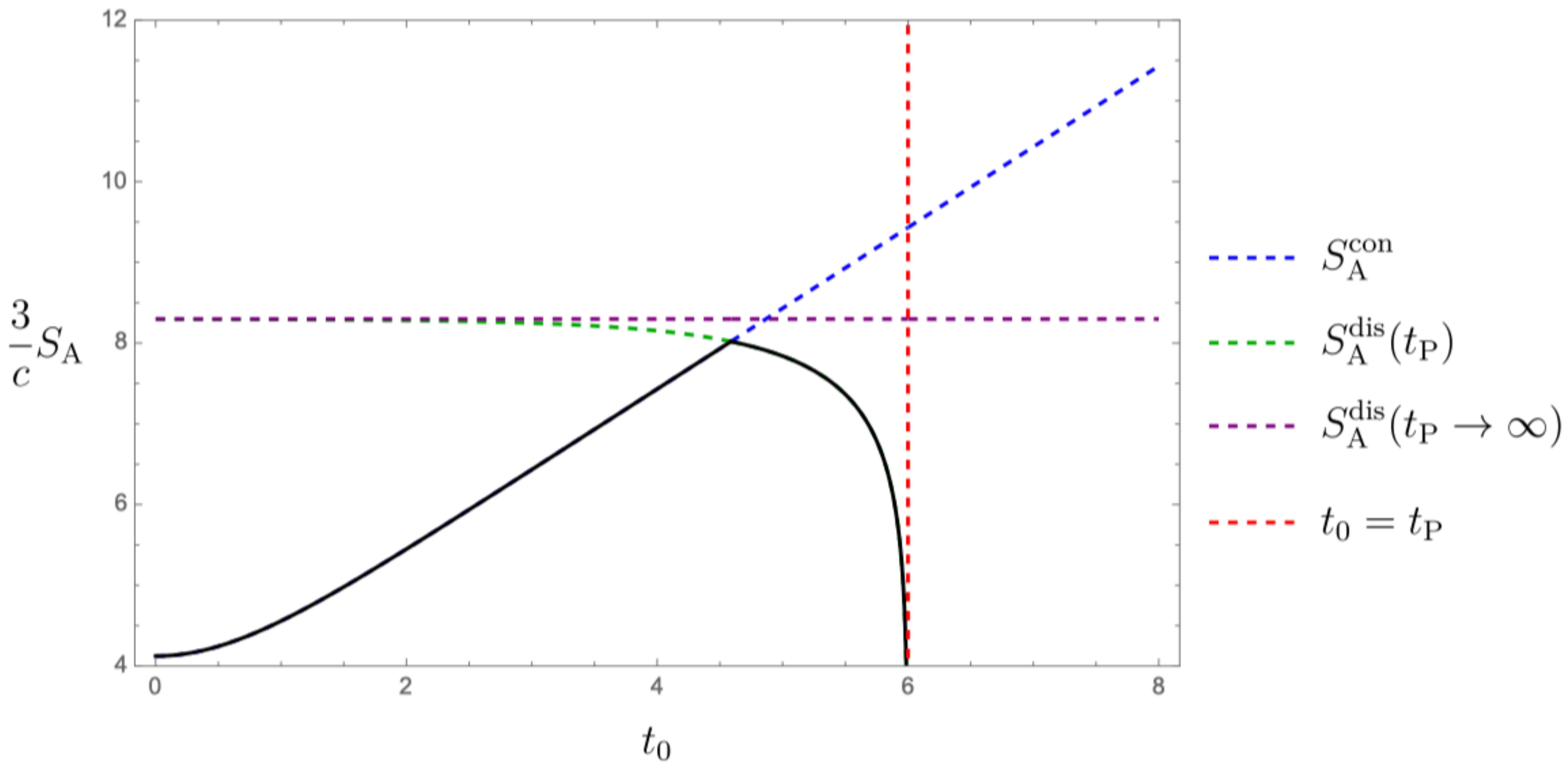
Perspective 02: Holographic Pseudo Entropy



Final State Projection at a finite time

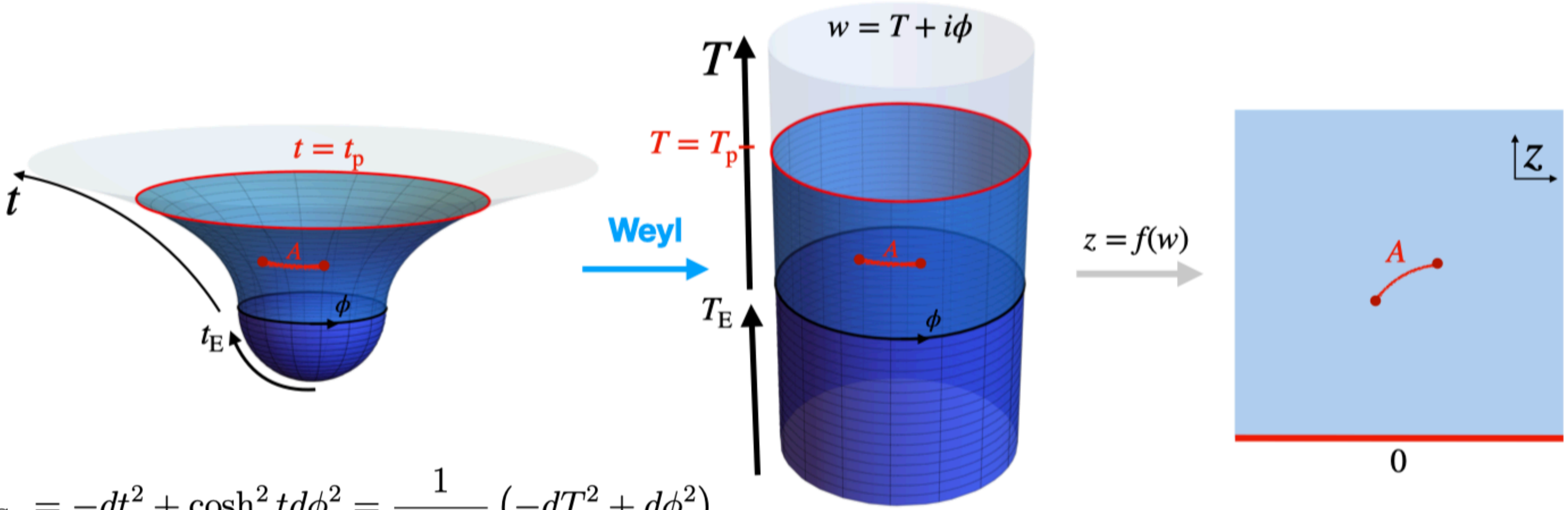
$$S_A^{\rm dis} = \frac{c}{3}\log\left[\frac{2}{\epsilon}\cdot\frac{\sinh t_{\rm P} - \sinh t_0}{\cosh t_{\rm P}}\right] - \frac{c}{3}\eta_*,$$

Boundary entropy



Perspective 02: Holographic Pseudo Entropy

Calculations from CFT_2 on dS_2



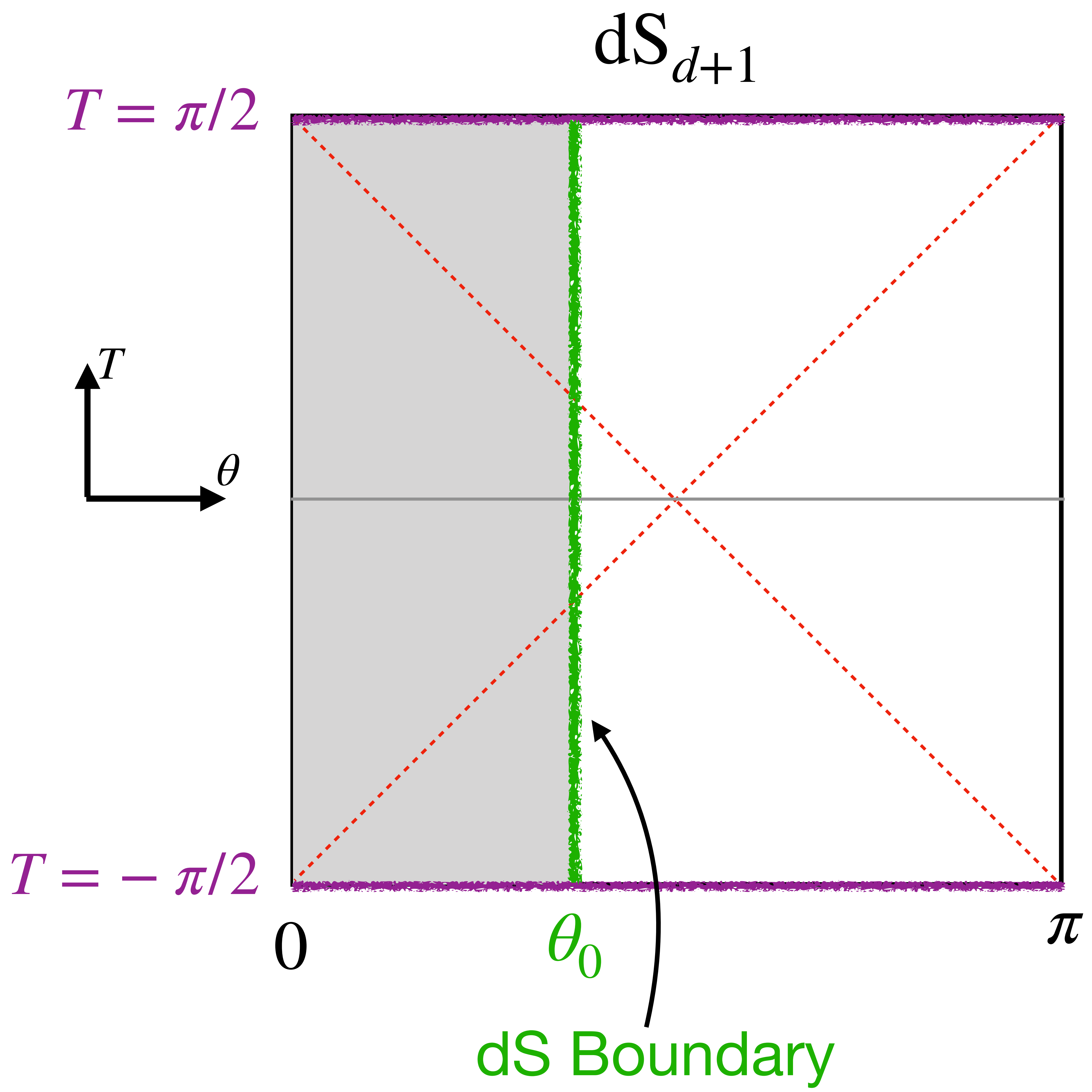
$$ds^2_{\mathrm{dS}_2} = -dt^2 + \cosh^2 t d\phi^2 = \frac{1}{\cos^2 T}\left(-dT^2 + d\phi^2\right)$$

$$S_A^{\mathrm{con,cyl}} = \frac{c}{6}\log\frac{|f(w_1) - f(w_2)|^2}{\epsilon^2 |f'(w_1)||f'(w_2)|}.$$

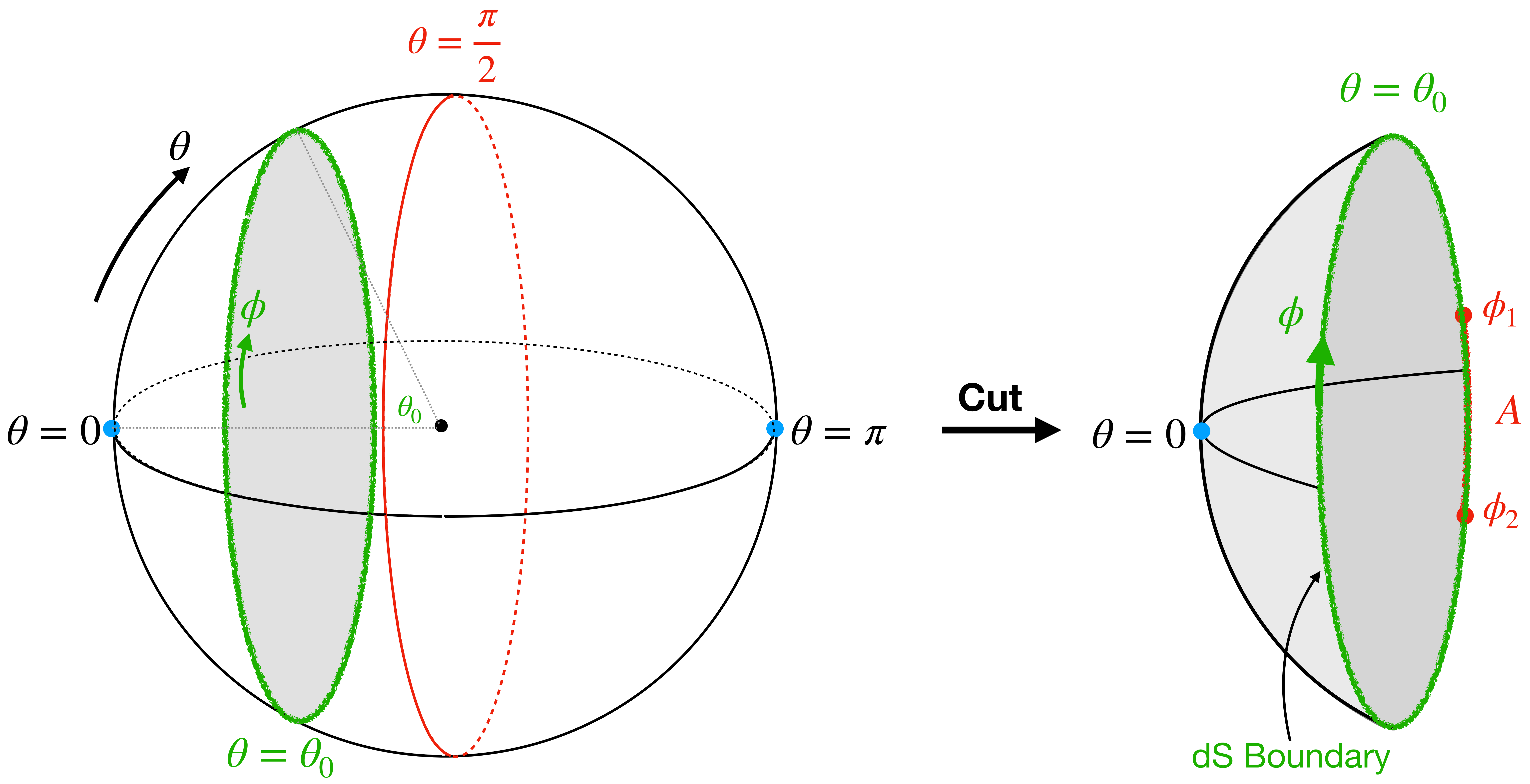
$$S_A^{\mathrm{dis,cyl}} = \frac{c}{6}\log\frac{|f(w_1) - \bar{f}(\bar{w}_1)||f(w_2) - \bar{f}(\bar{w}_2)|}{\epsilon^2 |f'(w_1)||f'(w_2)|} + 2S_{\mathrm{bdy}}.$$

03. Holography for a Half de Sitter

Topic Today: A half de Sitter space



Topic Today: A half de Sitter space



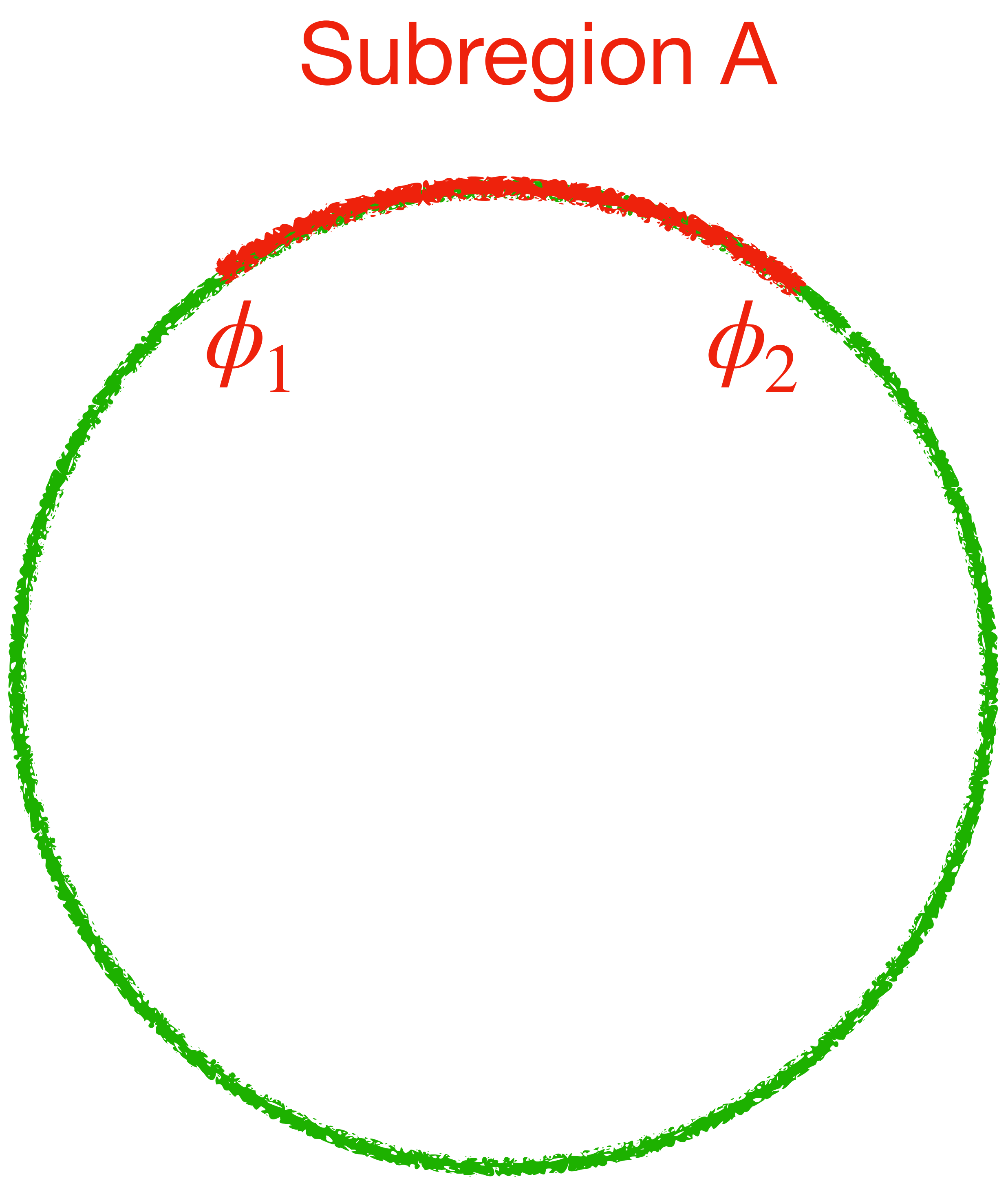
$$ds^2 = -dt^2 + \cosh^2 t\,(d\theta^2 + \sin^2\theta d\phi^2)$$

3-dim de Sitter bulk spacetime

$$ds^2 = -dt^2 + \cosh^2 t \sin^2\theta_0 d\phi^2$$

2-dim de Sitter spacetime

Perspective 01: Holographic Entanglement Entropy Without EOW brane



RT formula in dS

$$S_A = \frac{D_{12}}{4G_N} \qquad ?$$

Perspective 01: Holographic Entanglement Entropy Without EOW brane

Bulk spacetime dS_3 $$ds^2 = -dt^2 + \cosh^2 t(d\theta^2 + \sin^2\theta d\phi^2).$$

Extremal surface -> spacelike geodesics

Spacelike geodesics exist only for $$\cos^2\theta_0 + \sin^2\theta_0\cos(\phi_1 - \phi_2) \geq 1 - \frac{2}{\cosh^2 t_0}.$$

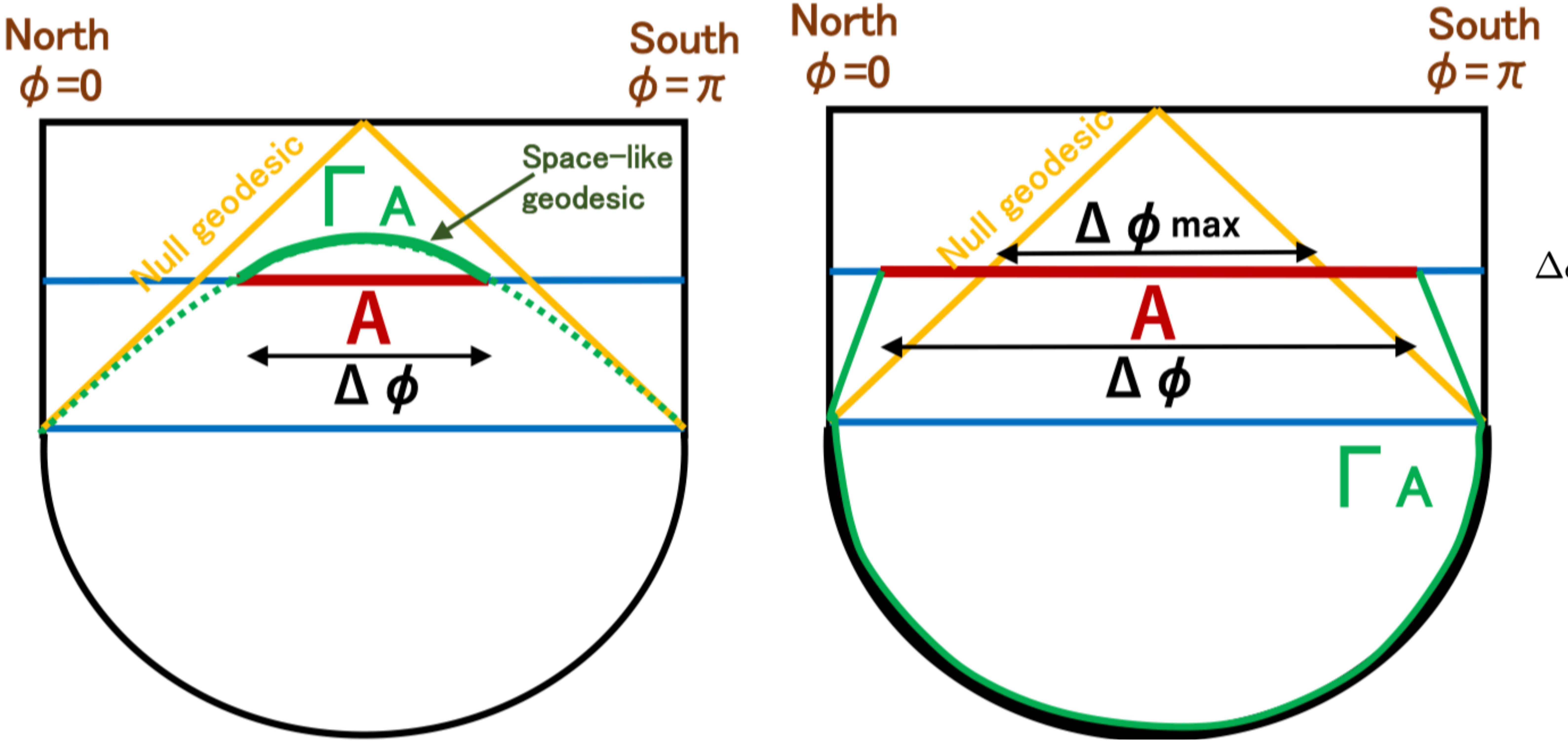
It is always violated at late times for any fixed subregions

Complex Geodesics Length/Area of Minimal Surface

$$D_{12} = \pi + i\mathrm{arccosh}\left[\sinh^2 t - \left(\cos^2\theta_0 + \sin^2\theta_0\cos(\phi_1 - \phi_2)\right)\cosh^2 t_0\right].$$

Perspective 01: Holographic Entanglement Entropy Without EOW brane

2-dim de Sitter Boundary at $\theta_0 = \pi/2$

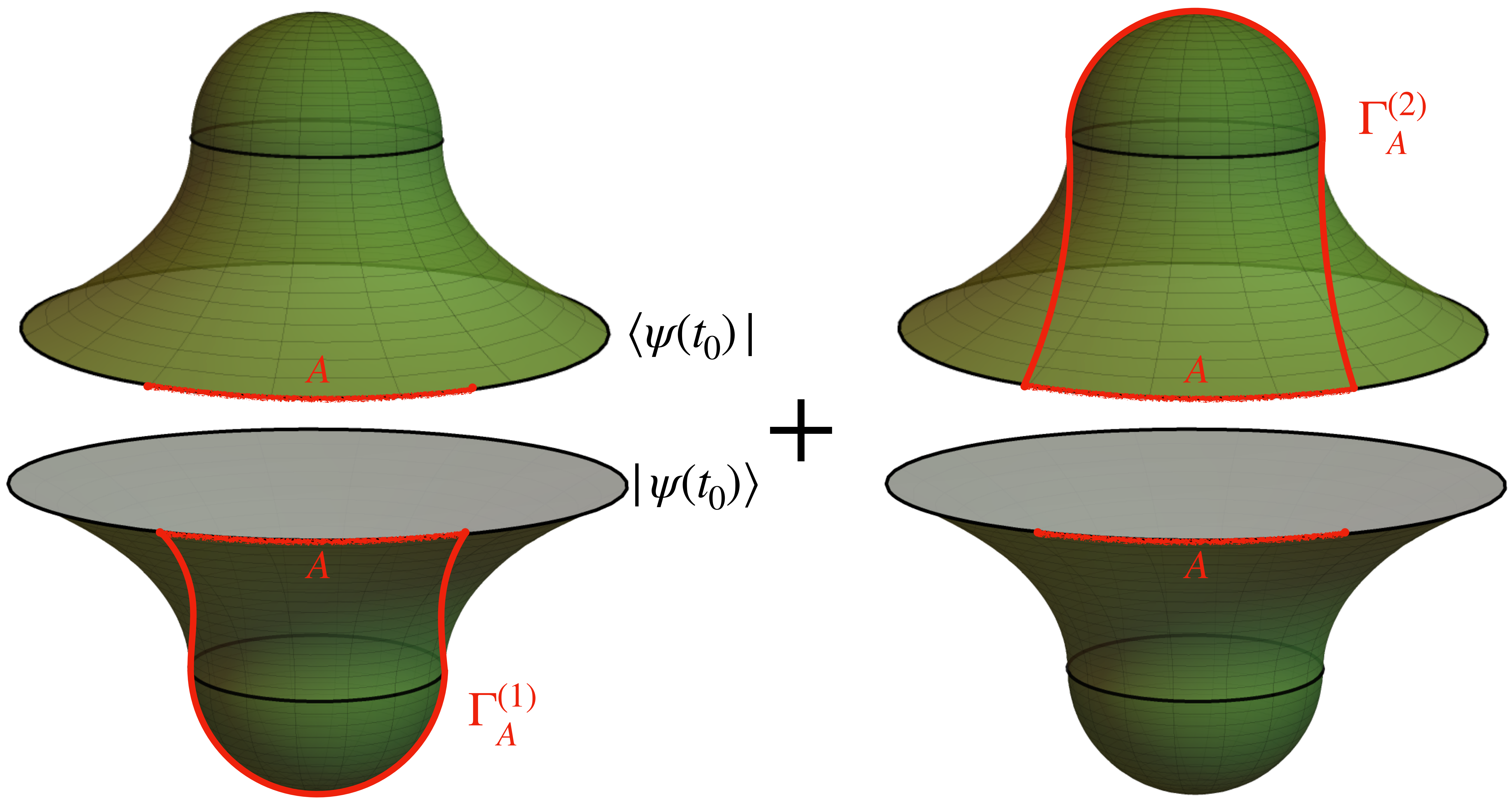


$$\Delta\phi_{\max} = \pi - 2\arctan\left[\sinh t_0\right].$$

$$D_{12} = \pi + i\,\mathrm{arccosh}\left[\sinh^2 t - \left(\cos^2\theta_0 + \sin^2\theta_0\cos(\phi_1-\phi_2)\right)\cosh^2 t_0\right].$$

Perspective 01: Holographic Entanglement Entropy Without EOW brane

There are two geodesics: in ket and bra geometry



$$Z_{\text{tot}}^{(n)} = e^{(1-n)S} + e^{(1-n)S^*},$$

$$S_A = \lim_{n\to 1} \frac{1}{1-n} \log Z_{\text{tot}}^{(n)} \simeq \frac{S+S^*}{2} = \text{Re}[S].$$

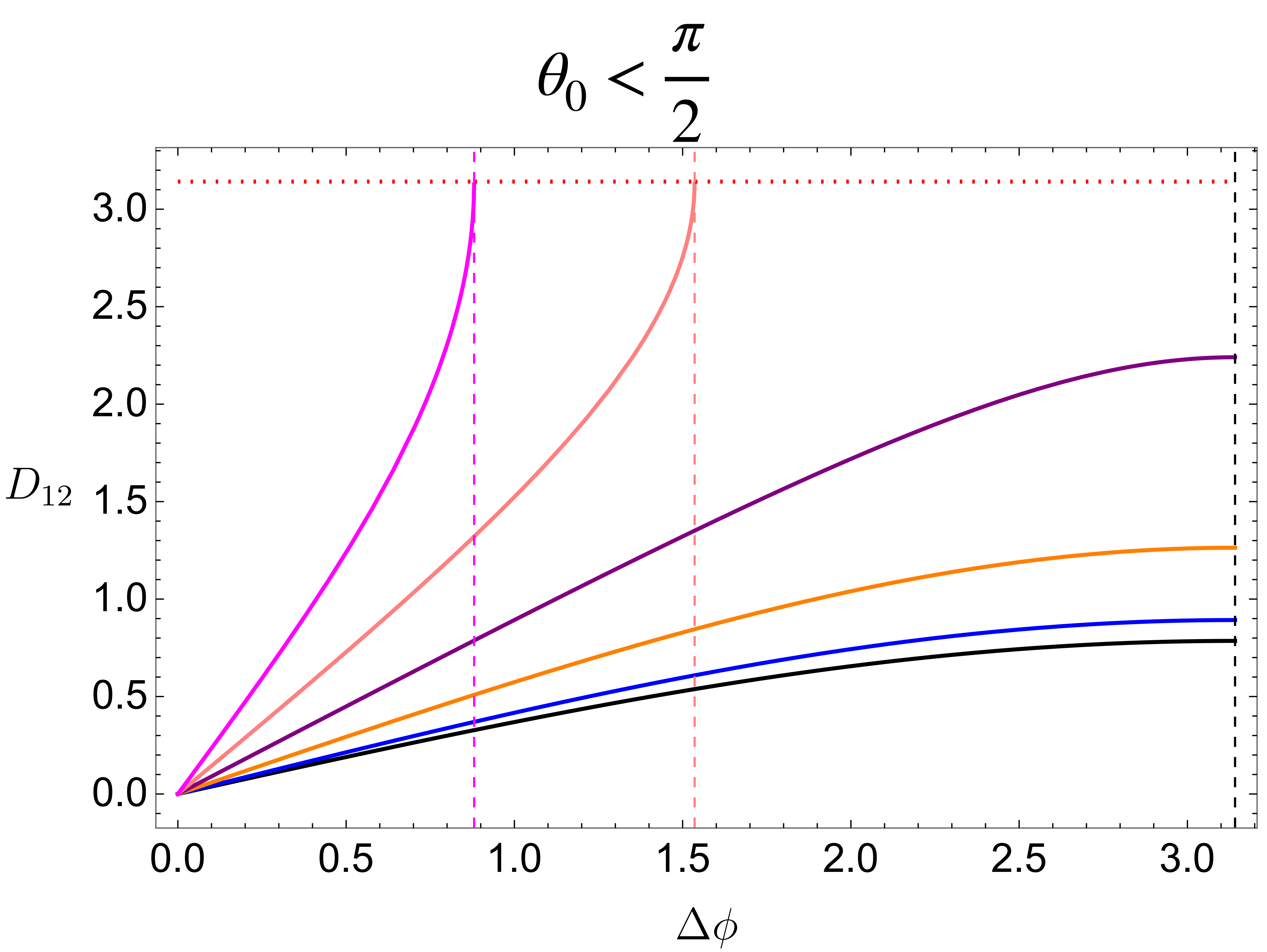
A half dS entropy

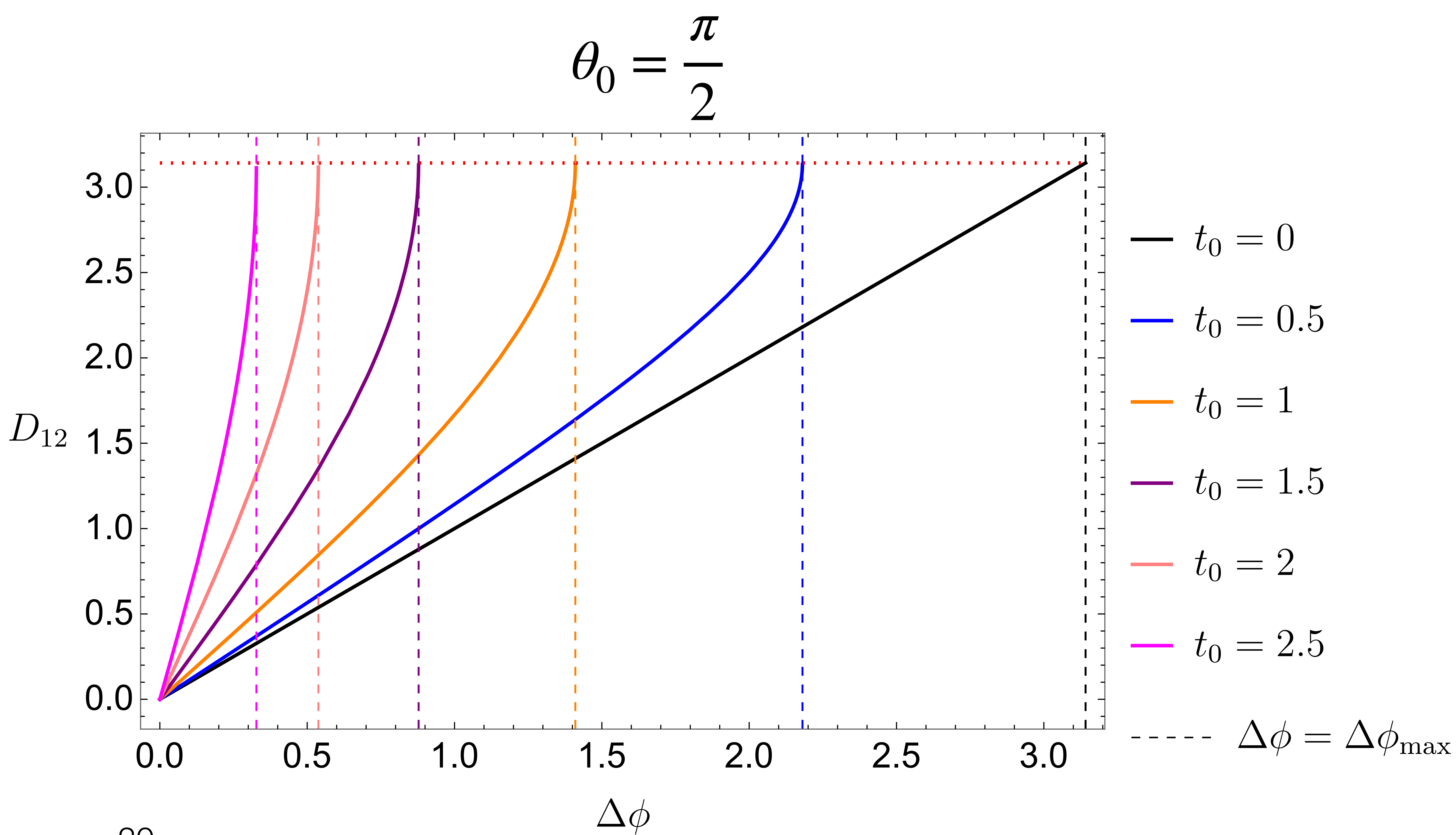
$$S_A = \frac{\pi}{4G_N} = \frac{1}{2} S_{\text{dS}},$$

Perspective 01: Holographic Entanglement Entropy Without EOW brane

Spacelike geodesics exist only for $$\cos^2\theta_0 + \sin^2\theta_0\cos(\phi_1 - \phi_2) \geq 1 - \frac{2}{\cosh^2 t_0}.$$

Geodesic Length at different time slices





Perspective 01: Holographic Entanglement Entropy Without EOW brane

But it is a convex function at late times!

$$\frac{d^2 D_{12}}{d\Delta\phi d\Delta\phi} = \frac{\sqrt{2}\cosh t \sin\theta_0 \sin\left(\frac{\theta_0}{2}\right)\left(\cosh^2 t \sin^2\theta_0 - 1\right)}{\left(1+\cos D_{12}\right)^{3/2}} > 0$$

$$t > t_{\text{max}} = \operatorname{arccosh}\left(\frac{1}{\sin\theta_0}\right) = \log\left(\cot\left(\frac{\theta_0}{2}\right)\right).$$

Intersection with cosmological horizon

Violation of (strong) subadditivity!

Perspective 01: Holographic Entanglement Entropy

Subadditivity $S_A + S_B \geq S_{AB}$

Strong Subadditivity (SSA) $S_{AB} + S_{BC} \geq S_{ABC} + S_B$

Violation of (strong) subadditivity!

↓

Boundary theory is highly non-local (but different from TTbar deformation)

Example: $$S = \int dx^d \phi(x) e^{(-\partial_x^2)^q} \phi(x), \qquad S_A \propto \left(\frac{L_A}{\epsilon}\right)^{d-2+2q},$$ **[Li, Takayanagi 2010]**

entanglement entropy defined from the replica method (Euclidean path-integral) is not guaranteed to satisfy the subadditivity!

Perspective 01: Holographic Entanglement Entropy

Subadditivity $S_A + S_B \geq S_{AB}$

Strong Subadditivity (SSA) $S_{AB} + S_{BC} \geq S_{ABC} + S_B$

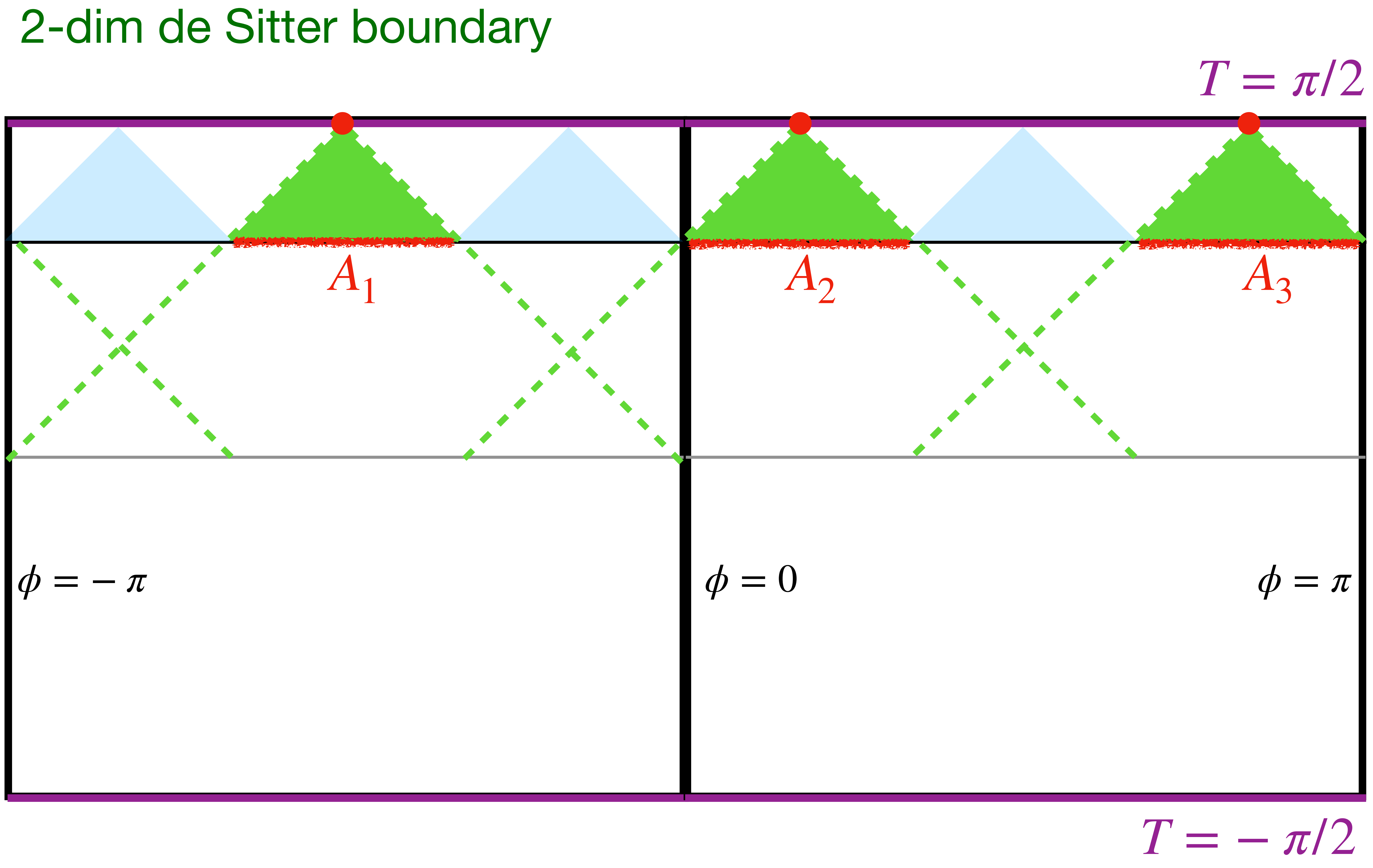
Violation of (strong) subadditivity!

↓

The different time slices are not just a unitary/isometric evolution in dS space

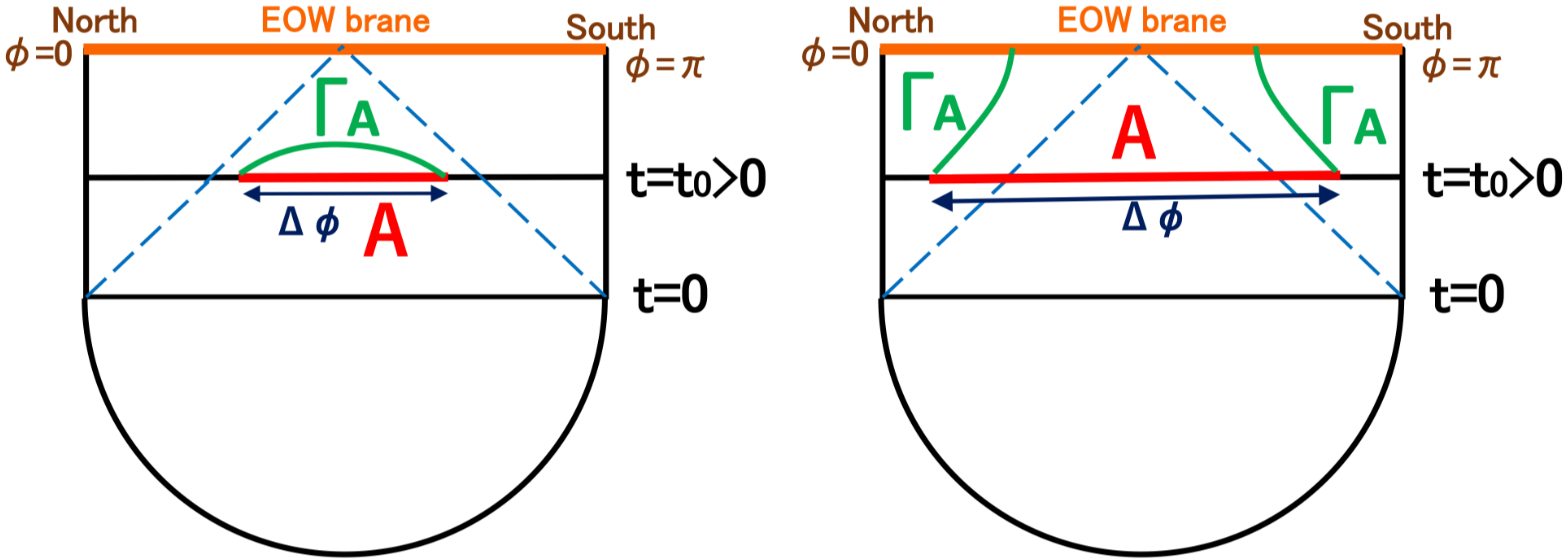
Perspective 01: Holographic Entanglement Entropy

Overcounting Hilbert Space



$$S_{A_1 \cup A_2 \cdots \cup A_N} = N \times \frac{1}{2} S_{\mathrm{dS}} \to \infty$$

Perspective 02: Holographic Pseudo Entropy with EOW brane



See our paper for more details :
Holographic pseudo entropy, higher dimensional de Sitter, different EOW branes

Thanks for your attention!